HIGH-TEMPERATURE ANYON SUPERCONDUCTIVITY

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Abstract

The screening of an applied magnetic field in a charged anyon fluid at finite density ($\mu \neq 0$) and temperature ($T \neq 0$) is investigated. For densities typical of high-temperature superconducting materials we find that the anyon fluid exhibits a superconducting behavior at any temperature. The total Meissner screening is characterized by two penetration lengths corresponding to two short-range eigenmodes of propagation within the anyon fluid.

Since the proposition by Laughlin and his collaborators[1],[2] that condensation of bosonic composites of *anyonic* quasiparticles could give rise to high-T_C superconductivity, a significant work has been done in this direction.

The anyon superconductivity at $T = 0$ has been investigated by many authors[2]-[7]. Crucial to the interpretation of anyon superconductivity at $T = 0$ was the exact cancellation between the bare and induced Chern-Simons terms in the effective action of the theory.

In a recent paper[8], it was stressed that a self-consistent treatment of many-particle systems (i.e. $n_e(\mu) \neq 0$) at zero temperature requires to consider the zero temperature statistical limit ($T \to 0$, $\mu \neq 0$) instead of the QFT ($T = 0$, $\mu = 0$) formulation. In that case it was proven[8] that, contrary to

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what occurs in quantum field theory ($T = 0$), superconductivity arises only in the case that an external electric field, transverse to the supercurrent in the plane of the two-dimensional fluid, is considered. This transverse electric field can be interpreted as simulating the possible effects of the transverse voltage created by vortices[4].

The possible realization of anyon superconductivity at $T \neq 0$ has also been extensively investigated[5]-[7],[9],[10]. At finite temperature, based on non-vanishing corrections to the induced Chern-Simons coefficient, some authors (see, ref. [6]) have concluded that the superconductivity is lost at $T \neq 0$. In contrast with this result, in refs. [7],[10] it was argued that the non-vanishing corrections to the induced Chern-Simons coefficient is numerically negligible at $T \sim 100\ ^\circ K$. On the other hand, the development of a pole $\sim \left(\frac{1}{\mathbf{k}^2}\right)$ at $T \neq 0$ in the polarization operator component Π_{00}, characteristic of the Debye screening in plasmas, was found[7],[10] as the main reason for the lack of a total Meissner effect in the charged anyon fluid at finite temperature. In these papers it was discussed how the appearance of this pole leads to a partial Meissner effect with a penetration which appreciably increases with temperature. In ref. [5], similar results were independently obtained. There, it was claimed that the anyon model fails to provide a good superconducting behavior at finite temperature. The reason is that three independent components of the magnetic interaction were obtained within the charged anyon fluid at $T \neq 0$. Two of finite-range and a third one of long-range which vanishes only at $T = 0$.

In the present paper, working in the self-consistent field approximation [5],[7],[8],[10], we will show that the charged anyon fluid exhibits a superconducting behavior at any temperature. This superconductivity is characterized by a total Meissner screening. The external magnetic field is damped within the anyon fluid by two characteristic lengths corresponding to two short-range eigenmodes of propagation. We emphazise that the asymptotic conditions ($x \to \infty$) for the zero components of the Maxwell and Chern-Simons gauge potentials, A_0 and a_0 respectively, have been crucial to obtain this behavior. These field components play the role of Lagrange multipliers in the Extended Hamiltonian of the Dirac formulation of this theory. As it is known, the restrictions in the allowed asymptotic behavior of the Lagrange multipliers have profound consequences for gauge field theories[11].

The approach we follow is to compute the finite temperature effective action in the self-consistent field approximation starting from the Lagrangian

density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{N}{4\pi}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + en_eA_0 + i\psi^\dagger D_0\psi - \frac{1}{2m}\left|D_k\psi\right|^2 + \psi^\dagger\mu\psi \quad (1)$$

of a 2+1 dimension charged fluid of non-relativistic electrons, ψ, coupled to two independent gauge fields, A_μ and a_μ, which represent the electromagnetic field and the Chern-Simons field respectively. The covariant derivative is given by $D_\nu = \partial_\nu + i\left(a_\nu + eA_\nu\right), \quad \nu = 0,1,2$. The charged character of the fluid is implemented through the chemical potential μ; n_e is a background neutralizing "classical" charge density. From the electric charge neutrality condition, it is found[7],[8] that the system ground state has a non-zero expectation value of the Chern-Simons magnetic field $\left(\overline{b} = \frac{2\pi n_e}{N}\right)$.

Integrating out the electron field, we obtain the effective action corresponding to the Lagrangian density (1),

$$\Gamma_{eff}\left(A_\nu, a_\nu\right) = -\frac{1}{4}F_{\mu\nu}^2 - \frac{N}{4\pi}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + en_eA_0 - \beta^{-1}\ln\det G^{-1}\left(A_\nu, a_\nu\right) \quad (2)$$

Here $G\left(A_\nu, a_\nu\right)$ denotes the exact fermion Green's function in the many-particle background (i.e. in the presence of $\overline{b}$ and μ), and β is the inverse absolute temperature.

To investigate the linear response of the medium to an applied external magnetic field, it is enough to consider small fluctuations of the gauge potentials around the many-particle ground state. That is, we can evaluate Γ_{eff} up to second order in these small quantities,

$$\Gamma_{eff}\left(A_\nu, a_\nu\right) = -\frac{1}{4}F_{\mu\nu}^2 - \frac{N}{4\pi}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + en_eA_0 + \Gamma^{(2)} \quad (3)$$

$$\begin{aligned}\Gamma^{(2)} &= \int dx\Pi_\nu\left(x\right)\left[a_\nu\left(x\right) + eA_\nu\left(x\right)\right] + \\ &+ \int dxdy\left[a_\nu\left(x\right) + eA_\nu\left(x\right)\right]\Pi_{\mu\nu}\left(x,y\right)\left[a_\nu\left(y\right) + eA_\nu\left(y\right)\right] \quad (4)\end{aligned}$$

$\Gamma^{(2)}$ is the fermion contribution to the effective action in the above approximation, Π_ν and $\Pi_{\mu\nu}$ represent the fermion tadpole and polarization operators respectively. An essential point in the study of this effective theory is the

calculation of these operators by using the fermion thermal Green's function defined in the presence of the background field $\bar{b}$. With this aim, we apply the Matsubara finite-temperature technique in the Landau gauge for the electromagnetic field A_μ, as well as for the Chern-Simons field a_μ, taking, as in ref. [8], the thermal Green's function in the small background density approximation ($n_e < Nm^2$). This approximation is satisfied within the order of the typical densities reported in high-temperature superconductivity[7].

The leading behavior of these operators for static ($k_0 = 0$) and slowly ($\mathbf{k} \sim 0$) varying configurations, and with the spatial momentum specialized in the frame $\mathbf{k} = (k, 0)$, are (the details of the calculations will be published elsewhere)

$$\Pi_k(x) = 0, \qquad \Pi_4(x) = -n_e \tag{5}$$

$$\Pi_{\mu\nu} = \begin{pmatrix} \mathit{\Pi_0} + \mathit{\Pi_0}'\,k^2 & 0 & \mathit{\Pi_1}\,k \\ 0 & 0 & 0 \\ -\mathit{\Pi_1}\,k & 0 & \mathit{\Pi_2}\,k^2 \end{pmatrix} \tag{6}$$

where

$$\mathit{\Pi_0} = -\frac{m}{4\pi}\left[\tanh\left(\frac{\beta\mu}{2}\right) + 1\right] \tag{7}$$

$$\mathit{\Pi_0}' = -\frac{\beta}{48\pi}\,\mathrm{sech}^2\left(\frac{\beta\mu}{2}\right) \tag{8}$$

$$\mathit{\Pi_1} = \frac{i\beta\bar{b}}{96\pi m}\,\mathrm{sech}^2\left(\frac{\beta\mu}{2}\right) \tag{9}$$

$$\mathit{\Pi_2} = \frac{1}{48\pi m}\left[\tanh\left(\frac{\beta\mu}{2}\right) + 1\right] \tag{10}$$

The operators (5), and (6) reduce to the ones reported in ref. [7] in the $T \to 0$ limit.

To investigate the linear response of the anyon fluid to an applied external magnetic field we have to find the extremum equations derived from the effective action (3). This formulation is what is known in the literature as the self-consistent field approximation[7],[10]. As our main interest is to investigate the behavior, within the anyon fluid, of an applied external magnetic field, we have to define a geometry for the medium. As usual,

we will consider a material confined to a semi-infinite plane $-\infty < y < \infty$ with boundary at $x = 0$. The external magnetic field will be applied from the vacuum ($-\infty < x < 0$). We restrict our solution to gauge field configurations which are static and uniform in the y-direction.

The corresponding Maxwell and Chern-Simons extremum equations are respectively,

$$\partial_\nu F^{\nu\mu} = eJ^\mu_{ind} \tag{11}$$

$$-\frac{N}{4\pi}\varepsilon^{\mu\nu\rho} f_{\nu\rho} = J^\mu_{ind} \tag{12}$$

Here, $f_{\mu\nu}$ is the Chern-Simons gauge field strength tensor, defined as $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and J^μ_{ind} is the 4-current density induced by the anyon system at finite temperature and density. Their different components are given by

$$J^4_{ind}(x) = \mathit{\Pi_0}\left[a_0(x) + eA_0(x)\right] + \mathit{\Pi_0}'\partial_x(\mathcal{E} + eE) + i\mathit{\Pi_1}(b + eB) \tag{13}$$

$$J^1_{ind}(x) = 0, \qquad J^2_{ind}(x) = i\mathit{\Pi_1}(\mathcal{E} + eE) + \mathit{\Pi_2}\partial_x(b + eB) \tag{14}$$

in the above expressions we used the following notation: $\mathcal{E} = f_{01}$, $E = F_{01}$, $b = f_{21}$ and $B = F_{21}$. Eqs. (13)-(14) play the role in the anyon fluid of the London equations in BCS superconductivity. When the induced currents (13), (14) are substituted in eqs. (11) and (12) we find, after some manipulation, the set of independent differential equations,

$$\omega\partial_x^2 B + \alpha B = \gamma\left[\partial_x E - \sigma A_0\right] + \tau\, a_0 \tag{15}$$

$$\partial_x B = \kappa\partial_x^2 E + \eta E \tag{16}$$

$$\partial_x a_0 = \chi\partial_x B \tag{17}$$

The coefficients appearing in these differential equations depend on the components of the polarization operators through the relations,

$$\omega = \frac{2\pi}{N}\mathit{\Pi_0}', \quad \alpha = ie^2\mathit{\Pi_1}, \quad \tau = -e\mathit{\Pi_0}, \quad \chi = -\frac{2\pi}{eN}, \quad \sigma = \frac{e^2}{\gamma}\mathit{\Pi_0}, \quad \eta = -\frac{ie^2}{\delta}\mathit{\Pi_1}$$

$$\gamma = 1 - e^2 \mathit{\Pi_0}' - \frac{2\pi i}{N} \mathit{\Pi_1}, \quad \delta = 1 + e^2 \mathit{\Pi_2} + \frac{2\pi i}{N} \mathit{\Pi_1}, \quad \kappa = -\frac{2\pi}{N\delta} \mathit{\Pi_2}, \qquad (18)$$

The extremum equations (15)-(17) are not essentially different from those found for the anyon effective theory at finite temperature by other authors[7],[10]. Distinctive of these equations is the appearance of the nonzero constant coefficients σ and τ. They are related to the Debye screening which is a property of any charged medium. It is a peculiar fact that in the anyon fluid these coefficients appear linked to the magnetic field (see eq. (15)). As a consequence, the zero components of the gauge potentials, A_0 and a_0, play a nontrivial role in the field equations for the magnetic field. Therefore, it is natural to expect that their contribution to the Meissner effect can be significant. As we will show below, to consider the proper asymptotic behavior of these potentials is crucial for the realization of the total Meissner screening in the charged anyon fluid.

To solve eqs. (15)-(17) we can conveniently arrange them to obtain,

$$a\partial_x^4 E + d\partial_x^2 E + cE = 0 \qquad (19)$$

where $a = \omega\kappa$, $d = \omega\eta + \alpha\kappa - \gamma - \tau\kappa\chi$, and $c = \alpha\eta - \sigma\gamma - \tau\eta\chi$. Then the solutions for the fields E, and B, and for the potentials a_0 and A_0, can be obtained from (19), (16), (17) and the definition of E in terms of A_0, respectively. Being (19) a higher order differential equation, its solution belongs to a wider class if compared to that corresponding to the original eqs. (15)-(17). Thus, to exclude the redundant solutions we have to require that they satisfy eq. (15) as a supplementary condition. In this way we can reduce the number of independent unknown coefficients to six, which is the number corresponding to the original system (15)-(17).

Solving eq. (19) we obtain,

$$E\left(x\right) = C_1 e^{-x\xi_1} + C_2 e^{x\xi_1} + C_3 e^{-x\xi_2} + C_4 e^{x\xi_2}, \qquad (20)$$

where

$$\xi_{1,2} = \left[-d \pm \sqrt{d^2 - 4ac}\right]^{\frac{1}{2}} / \sqrt{2a} \qquad (21)$$

take real values at any temperature when evaluated with the typical values $n_e = (1 \sim 5) \times 10^{14} cm^{-2}$, $m = 2m_e$ ($m_e = 2.6 \times 10^{10} cm^{-1}$ is the electron mass) and $|N| = 2$.

With the solution (20), eqs. (16), (17), and the definition $E = -\partial_x A_0$, we find,

$$B(x) = \frac{\xi_1^2\kappa + \eta}{\xi_1}\left(C_1 e^{-x\xi_1} - C_2 e^{x\xi_1}\right) + \frac{\xi_2^2\kappa + \eta}{\xi_2}\left(C_3 e^{-x\xi_2} - C_4 e^{x\xi_2}\right) + C_5 \quad (22)$$

$$a_0(x) = \chi\frac{\xi_1^2\kappa + \eta}{\xi_1}\left(C_1 e^{-x\xi_1} - C_2 e^{x\xi_1}\right) + \chi\frac{\xi_2^2\kappa + \eta}{\xi_2}\left(C_3 e^{-x\xi_2} - C_4 e^{x\xi_2}\right) + C_6 \quad (23)$$

$$A_0(x) = \frac{1}{\xi_1}\left(-C_1 e^{-x\xi_1} + C_2 e^{x\xi_1}\right) + \frac{1}{\xi_2}\left(-C_3 e^{-x\xi_2} + C_4 e^{x\xi_2}\right) + C_7 \quad (24)$$

The extra unknown coefficient is eliminated, as it was explained above, substituting the solutions (20), (22), (23) and (24) into eq. (15) to obtain the relation,

$$C_5 = \frac{\tau}{\alpha}C_6 + \frac{\sigma\gamma}{\alpha}C_7 \quad (25)$$

The last relation establishes a connection between the asymptotic conditions for the zero components of the gauge potentials and the asymptotic condition for the magnetic field.

To determine the six independent coefficients we have to consider the proper boundary conditions for the possible realization of the Meissner effect. Thus, we consider that the magnetic field has a boundary value $B(x=0) = \overline{B}$ and is finite when $x \to \infty$, that the boundary value of the electric field is zero, $E(x=0) = 0$, and that the asymptotic values of the zero component of the gauge potentials are zero, i.e. $A_0(x \to \infty) = 0$ and $a_0(x \to \infty) = 0$. These asymptotic conditions for the gauge potentials guarantee well behaved Poisson brackets between the dynamical variables of the theory and the generators of the gauge transformations[11].

With the above conditions we obtain that $C_2 = C_4 = C_5 = C_6 = C_7 = 0$ and $C_1 = -C_3$, where C_1 depends on the magnetic field boundary value and temperature through the relation,

$$C_1 = \frac{\xi_1\xi_2}{\left(\xi_2^2\kappa + \eta\right) - \left(\xi_1^2\kappa + \eta\right)}\overline{B} \quad (26)$$

The magnetic field penetration (i.e. at $x \geq 0$) is then given by,

$$B(x) = B_1(T)\ e^{-x\xi_1} + B_2(T)\ e^{-x\xi_2} \tag{27}$$

where the temperature dependent coefficients are,

$$B_1(T) = \frac{\xi_2\left(\xi_1^2\kappa+\eta\right)}{\left(\xi_1^2\kappa+\eta\right)-\left(\xi_2^2\kappa+\eta\right)}\overline{B}, \qquad B_2(T) = \frac{\xi_1\left(\xi_2^2\kappa+\eta\right)}{\left(\xi_2^2\kappa+\eta\right)-\left(\xi_1^2\kappa+\eta\right)}\overline{B} \tag{28}$$

Hence, we have that within the anyon fluid the applied magnetic field falls down exponentially on two essentially different scales, $\lambda_1 = 1/\xi_1$ and $\lambda_2 = 1/\xi_2$, which characterize two eigenmodes of propagation inside the fluid. If we define the effective penetration length, $\overline{\lambda}$, as the distance x where the magnetic field falls down to a value $B\left(\overline{\lambda}\right)/\overline{B} \simeq 0.38$, we find that at $T \approx 200K$ the screening is complete at $\overline{\lambda} \approx 8.5 \times 10^{-11}\ cm$. Considering the obtained values for the C_i's coefficients in the solution (20), we also find that the induction of an electric field is intimately linked to the Meissner effect in the anyon fluid.

Finally, a few comments are in order. We want to point out that a partial Meissner solution[7],[10], would imply a variation in the asymptotic conditions of the gauge potentials. Specifically, it can be obtained for $A_0(x \to \infty) = \overline{D}_1$ and $a_0(x \to \infty) = \overline{D}_2$, ($\overline{D}_1$ and $\overline{D}_2$ constants). In statistical gauge theory these constant asymptotic field configurations are not gauge equivalent (under proper, periodic gauge transformations) to the trivial vacuum[12]. To study the problem under these nontrivial asymptotic conditions we have to reconsider the whole gauge formulation of the theory taking into account the possible appearance of an overall "charge rotation"[11]. An important point that we will discuss in detail in a future publication is the existence of some indications, within the present approximation, of possible phase transitions from the superconducting phase to the normal one.

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